A single spectral line → splits up into three components → Normal Zeeman Effect

More complicated splitting → Anomalous Zeeman effect
↓
introduction of spin of electron in vector atom model.

l^* and s^* vectors precess around their resultant vector j^* (total angular momentum vector)

⇒

$$j^* = l^* + s^* \quad \text{---(1)}$$

The magnetic moment due to orbital motion

$$\mu_l = l^* \frac{eh}{4\pi m_0} \quad \text{---(2)}$$

μ_l directed oppositely to l^* due to → negative charge of electron.

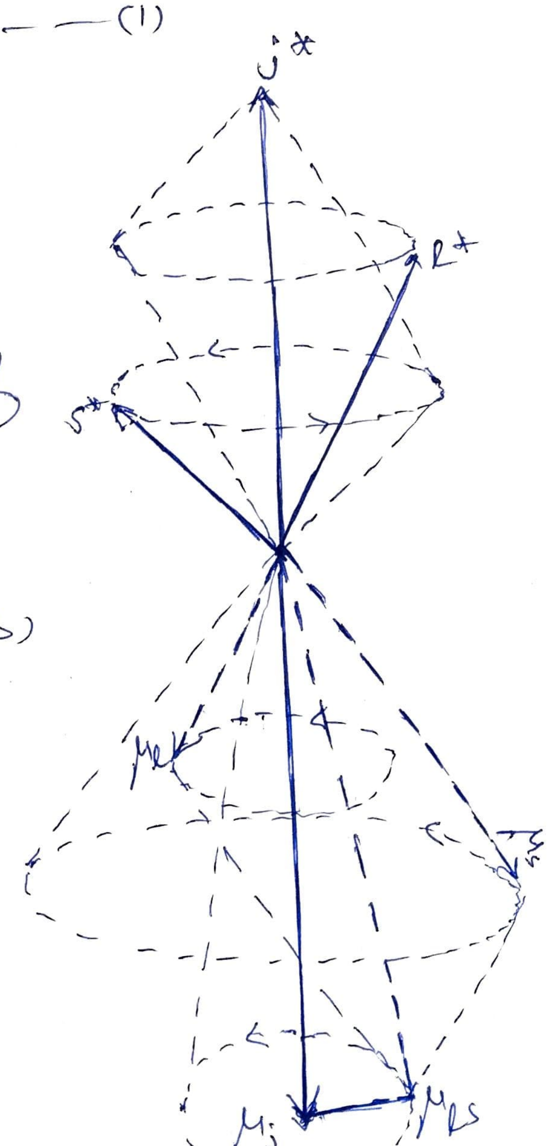
Magnetic moment due to spin

⇒ $$\mu_s = 2s^* \frac{eh}{4\pi m_0} \quad \text{---(3)}$$

and directed opposite to s^*.

l^* and s^* precess around j^*, μ_l and μ_s will also precess around μ_j

To be noted that μ_j is not in the line of the resultant of μ_l and μ_s.



The Precession of mechanical and magnetic moments

To find → The resultant magnetic moment of the electron, we resolve μ_{ls} into two directions (i) along j^* and (ii) perpendicular to it.

The perpendicular component → average out to zero.

Parallel component is added up. (continuous change of direction)

μ_j = Component of μ_l along this direction of j^* + Component of μ_s along the direction of j^*

$$\mu_j = \frac{eh}{4\pi m_0} l^* \cos(l^* j^*) + \frac{eh}{4\pi m_0} \cdot 2s^* \cos(s^* j^*)$$

Bohr magneton ←

$$= \frac{eh}{4\pi m_0} \left[l^* \cos(l^* j^*) + 2s^* (\cos s^* j^*) \right] \quad \text{— (4)}$$

~~$\mu_j = j^* g$~~ The first term → equivalent to one Bohr magneton, the quantity determined by the bracket gives the total magnetic moment of the atom in Bohr magnetons

This bracket term is readily evaluated by setting it equal to j^* times a constant 'g',

$$j^* \cdot g = l^* \cos(l^* j^*) + 2s^* \cos(s^* j^*)$$

according to the cosine law (5) and vector model

$$s^{*2} = l^{*2} + j^{*2} - 2 l^* j^* \cos(l^* j^*) \quad \text{— (6)}$$

we obtain

$$l^* \cos(l^* j^*) = \frac{j^{*2} + l^{*2} - s^{*2}}{2j^*} \quad \text{--- (7)}$$

Similarly

$$s^* \cos(s^* j^*) = \frac{j^{*2} - l^{*2} + s^{*2}}{2j^*} \quad \text{--- (8)}$$

Substituting these eq^ns in eq^n (5)

$$j^* g = \left[\frac{j^{*2} + l^{*2} - s^{*2}}{2j^*} + \frac{j^{*2} + s^{*2} - l^{*2}}{j^*}\right]$$

$$g = 1 + \frac{j^{*2} + s^{*2} - l^{*2}}{2j^{*2}} \quad \text{--- (9)}$$

in terms of quantum numbers l, j and spin s

Exactly same as given by Landé's model

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \quad \text{--- (10)}$$

If now the atom is placed in the weak magnetic field, the total angular momentum vector j^* precesses around the B direction.

Result → change in the energy of electron by an amount

From eq^n (4)

The ratio of the total magnetic and mechanical moments of the atom, μ_j and p_j is just

$$\frac{\mu_j}{p_j} = g \cdot \frac{e}{2m_0} \quad \text{--- (11)} \qquad \text{where } p_j = \frac{j^* h}{2\pi}$$

Due to the electron's anomalous spin magnetic moment, s^* tends to precess twice as fast around B (or H) as does l^*.
If the field is not too strong the coupling between l^* and s^* is sufficiently strong to maintain a constant j^*, so that this resultant precesses with a compromise angular velocity, by Larmor theorem, given by g times the orbital precession angular velocity

$$\omega_L = B g \frac{e}{2m_0} \quad (12)$$

The total energy of the precession is given by the precessional angular velocity ω_L times the component of the resultant mechanical moment $j^* h/2\pi$ on the axis of rotation B.

$$\Delta W = \omega_L j^* \frac{h}{2\pi} \cos(j^* B)$$

$$= B \cdot g \frac{e}{2m_0} j^* \frac{h}{2\pi} \cos(j^* B) \quad (13)$$

In terms of the magnetic quantum number m, $j^* h/2\pi \cdot \cos(j^* B)$ is just equal to $m_j h/2\pi$

So that

$$\Delta W = B \cdot g \frac{e}{2m_0} \cdot m_j \frac{h}{2\pi} = m_j \cdot g \cdot B \cdot \frac{eh}{4\pi m_0}$$

$$(14)$$

$$\Delta \nu = m_j \, g \, \frac{eB}{4\pi m_0}$$

$$\Delta \bar{\nu} = m_j \cdot g \cdot \frac{Be}{4\pi m_0 c} \ \text{cm}^{-1} \quad \text{--- (15)}$$

with $g = 1$ this eq^n reduces to Lorentz's classical formula.

Since the field H is the same for all levels of a given atom, it is convenient to express the Zeeman splitting in terms of Lorentz Unit

$$L = \frac{Be}{4\pi m_0 c}$$

$$\Rightarrow \quad \Delta \bar{\nu} = m_j \cdot g \cdot L \ \text{cm}^{-1} \quad \text{--- (16)}$$

The change in energy for each m_j level from the original level, and the shift is proportional to the field strength B. With $m_j = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots \pm j$, the g factor is seen to be of primary importance in splitting of the each level.

Consider, for example, the splitting of a $^2P_{3/2}$ level in a weak magnetic field. With $j = 3/2$, there are four magnetic levels $m_j = 3/2, 1/2, -1/2, -3/2$ shifted from the field-free level by $m_j g = 6/3, 2/3, -2/3$ and $-6/3$.